Eur. Phys. J. D **6**, 179–183 (1999)

THE EUROPEAN PHYSICAL JOURNAL D

EDP Sciences

Electrodynamic trap for neutral atoms

E. Peik[a]

Max-Planck-Institut für Quantenoptik and Sektion Physik der Ludwig-Maximilians-Universität München
Hans-Kopfermann-Str. 1, 85748 Garching, Germany

Received: 7 August 1998 / Received in final form: 7 January 1999

Abstract. An electrodynamic trap is proposed that stores cold neutral atoms or nonpolar molecules in their ground state as well as in excited states by means of the quadratic Stark effect. The trap uses an oscillating hexapole field and a superposed static homogeneous field. The dynamics of an atom in this trap can be described as a harmonic oscillation in a static pseudopotential. Stability criteria and sample parameters for a number of atomic species are given.

PACS. 32.80.Pj Optical cooling of atoms; trapping – 32.60.+i Zeeman and Stark effects

1 Introduction

Traps for atoms and ions have become an important experimental tool. They allow long interaction times and therefore potentially high precision in spectroscopic measurements. Experiments can be performed with a fixed and eventually very small number of particles — finally opening the possibility to study the quantum behaviour of a single atom. In combination with techniques to cool the stored particles, traps have become the source of new states of matter like strongly coupled crystalline plasmas, de Broglie waves of long coherence length and Bose-Einstein condensates of weakly interacting gases.

Traps for neutral atoms [1–3] can be constructed using inhomogeneous electric or magnetic fields. The field shifts the atomic energy levels and the spatial dependence of this shift leads to a force acting on the center of mass motion of the atom. Since the ground state is always lowered by an external perturbation, a ground state atom will be attracted by a field maximum, it is a so-called high field seeker. Some of the higher lying states will be raised by the perturbation, and atoms in these levels consequently are low field seekers, *i.e.* attracted by a field minimum. Maxwell's equations do not permit a maximum of the modulus of a static electric or magnetic field in a region void of charges or currents [4]. This makes the realisation of a static electromagnetic trap for ground state atoms impossible. It can even be shown that no combination of static electric, magnetic and gravitational fields can produce a stable trap for ground state atoms [5].

While a maximum of the modulus of the electromagnetic field is not allowed by Maxwell's equations, a minimum can easily be generated. The magnetic or the electric quadrupole field — realized using two coils in anti-Helmholtz arrangement or the three hyperbolic electrodes of the Paul trap, respectively — possesses a point of vanishing field strength at the center. Consequently, it can be used to construct an electrostatic [6] or a magnetostatic trap [7] for atoms in low field seeking excited states. These traps are unstable against spontaneous decay of the excited state. Even if the atoms are in metastable states with long lifetime, they can suffer from inelastic exothermal collisions. Since the energy released in such a collision is at least of the order of magnitude of the trap depth, this leads to a loss mechanism that limits the storage time. Magnetic traps for low field seekers have been used with great success in the experiments demonstrating Bose-Einstein-condensation (BEC) of rubidium, lithium and sodium vapor (see [1] and references therein). In the case of cesium, however, the exothermal collisions have prevented the attainment of BEC in a static magnetic trap [8].

Traps for high field seekers can only be realized using time dependent fields. A magnetodynamic trap has been proposed to achieve BEC in hydrogen [9] and has been tested experimentally with laser cooled cesium atoms [10]. A nearly homogeneous static field was superposed on an inhomogeneous oscillating field to create — similar to the Paul trap for ions — a saddle point of the potential that periodically interchanges the stable and unstable direction. This principle will also be used in the electrodynamic hexapole trap proposed here. Magnetic traps make use of the linear Zeeman effect and are generally deeper than electric traps, that interact with atoms in the ground state only via the quadratic Stark effect. Strong alternating electric fields, however, are easier to produce than their magnetic counterparts, since in the latter case one has to worry about losses through ohmic resistances and eddy currents. A further advantage of an electric trap is that in the scalar Stark potential all Zeeman sublevels of the ground state can be trapped and remain degenerate.

[a] e-mail: peik@mpq.mpg.de

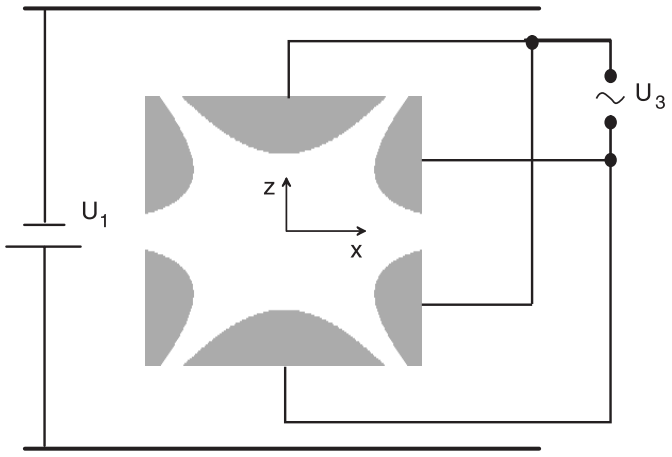

Fig. 1. Schematic of the electrodynamic trap resulting from the superposition of an alternating hexapole field and a static homogeneous field. The trap, shown here in a cut, is rotationally symmetric around the vertical z-axis. Gravity is assumed to act in negative z-direction. If the axial field of the hexapole is parallel to the homogeneous field, the quadratic Stark potential is radially stable, if the two fields are antiparallel, the potential is axially stable. Changing the sign of the hexapole potential periodically leads to dynamic stabilisation of the trapped atom.

This should make it possible to apply laser cooling methods, that often lead to a redistribution of the population between the Zeeman sublevels. Dynamic stabilisation of high field seekers in an electric field was first discussed by Shimizu and Morinaga [11,12] and an experiment demonstrating the focussing of a beam of metastable neon atoms using an oscillating electric two-phase quadrupole field has been performed [13].

The interaction of the atom with the electric field is given by the Stark potential $U_{\rm St} \propto \vec{E}^2$, one Cartesian component of the force by $F_x \propto \vec{E}\mathrm{d}\vec{E}/\mathrm{d}x$. To obtain a force of alternating sign one has to change the sign either of $\vec{E}$ or of $\mathrm{d}\vec{E}/\mathrm{d}x$, but not both signs simultaneously. This can be done by superposing a homogeneous static field and an inhomogeneous alternating field. For a three-dimensional trap, the most convenient inhomogeneous field can be looked for among the spherical multipoles. Using a singly periodic alternating field, the lowest order multipole, that fulfills the requirements of a stable electrodynamic trap, is the hexapole. The use of potentials with more complicated temporal dependences creates further possibilities. The trap proposed in [11,12] consists of three dipoles connected to an alternating three-phase voltage.

2 Equations of motion

The rotationally symmetric electric hexapole field is produced by two ring electrodes and two endcaps, each of which is connected to one of the rings. A cut through the xz-plane shows six poles (cf. Fig. 1). If the voltage U_3 is applied between the two pairs of electrodes, the electric potential is given by

$$\phi_3(x,y,z) = \frac{U_3}{4z_0^3}(2z^3 - 3zx^2 - 3zy^2)\,, \tag{1}$$

where $2z_0$ is the distance between the endcaps. To this we add a homogeneous field in the z-direction, generated by the dipole potential

$$\phi_1(z) = \frac{U_1 z}{2z_0}\,. \tag{2}$$

We suppose that the voltage U_1 is also applied across the characteristic length $2z_0$ of the hexapole. Furthermore, we introduce a weak quadrupole potential ϕ_2. This is not necessary for the functioning of the trap, but it opens the possibility to exert a constant force on the atoms, which can be used to balance the effect of gravity. The quadrupole potential is given by

$$\phi_2(x,y,z) = \frac{U_2}{4z_0^2}(2z^2 - x^2 - y^2)\,. \tag{3}$$

From the sum of the potentials one can easily calculate $\vec{E}$ and E^2. From the Stark potential $U_{\rm St} = -\alpha E^2/2$, where α denotes the static polarisability, the forces on the atom are derived:

$$F_x = \frac{\alpha}{2}\left(\frac{U_2^2 x}{2z_0^4} + \frac{9U_3^2(x^3+xy^2)}{4z_0^6} - \frac{3U_1U_3x}{2z_0^4}\right) \tag{4}$$

F_y can be obtained by exchanging x against y in the expression for F_x. The force in the z-direction reads

$$F_z = \frac{\alpha}{2}\left(\frac{2U_2^2 z}{z_0^4} + \frac{9U_3^2 z^3}{z_0^6} + \frac{U_1U_2}{z_0^3} + \frac{3U_1U_3z}{z_0^4} + \frac{9U_2U_3z^2}{z_0^5}\right). \tag{5}$$

Let us consider F_x first: The first two terms are always defocussing, independent of the sign of the voltages. They are, however, negligibly small with respect to the third term under the assumptions

$$(x^2+y^2+z^2) \ll z_0^2 \quad \text{and} \quad U_2 \ll U_1 \approx U_3\,. \tag{6}$$

We assume here that the atom is close to the center of the trap and that the quadrupole contribution is much smaller than the dipole and the hexapole. If an alternating voltage

$$U_3 = U_{30}\sin\omega t\,, \tag{7}$$

is used for the hexapole potential, the third term in F_x gives a force of alternating sign that is linear in x. This force is responsible for the stabilisation of the trap. In the expression for F_z the dominant contribution that gives rise to the dynamic trap is the fourth term, proportional to U_1U_3. The first two terms are again defocussing, but small under the assumptions (6). The third term $\propto U_1U_2$ produces the constant force for the compensation of gravity. For this purpose the voltage U_2 has to be adjusted to the value

$$U_2 = \frac{2mgz_0^3}{\alpha U_1} \tag{8}$$

which, as we will see, is much smaller than U_1 for typical parameters. The last term $\propto U_2U_3$ also creates a dynamic trap, but one that is anharmonic. However, since the quadrupole contribution is much weaker than the dipole ($U_2 \ll U_1$), this force is negligible.

Concentrating on the dominant terms $\propto U_1U_3$ due to the superposition of homogeneous field and hexapole, the equation of motion for the z-component is

$$\frac{\partial^2 z}{\partial \tau^2} - 2q_z \cdot \sin 2\tau \cdot z = 0\,, \tag{9}$$

with dimensionless parameters

$$\tau = \frac{\omega t}{2} \quad \text{and} \quad q_z = \frac{3\alpha U_1 U_{30}}{m\omega^2 z_0^4}\,. \tag{10}$$

This is the well-known Mathieu differential equation, which possesses stable solutions as long as $|q_z| < 0.907$. The equations of motion in x- and y-directions are of analogous form with $q_x = q_y = -q_z/2$. The dynamics of a trapped atom at the center of the hexapole trap follows the same laws as that of an ion in a Paul trap.

If $q_z^2 \ll 1$ one can — like in radiofrequency ion traps — use an adiabatic approximation [14], that separates the dynamics of the trapped particle into the driven micromotion at frequency ω and the slower secular motion in a static pseudopotential. The result for the pseudopotential of the hexapole trap is

$$\Psi(x,y,z) = \frac{\vec{F}^2}{4m\omega^2} = \frac{9\alpha^2 U_1^2 U_{30}^2}{64 z_0^8 m\omega^2}(x^2+y^2+4z^2) \tag{11}$$

$$= \frac{1}{64} q_z^2 m\omega^2 (x^2+y^2+4z^2)\,. \tag{12}$$

The secular motion is a harmonic oscillation with frequencies $\omega_{x,y} = q_z\omega/2\sqrt{8}$ in the radial and $\omega_z = q_z\omega/\sqrt{8}$ in axial direction.

3 Experimental considerations

The atomic parameter that determines the steepness of the pseudopotential is the ratio of the square of the polarisability over the mass. Of all ground state atoms, this quantity is maximal for lithium, which consequently is the most favorable candidate for an experimental realisation of the hexapole trap. Possible parameters are given in the first column of Table 1.

To judge the stability of the trap, the complete equations of motion were integrated numerically, including the defocussing terms in equations (4) and (5), with gravity compensated by the choice of U_2 according to equation (8). A typical trajectory calculated with the parameters for ^{7}Li from Table 1 is shown in Figure 2. It shows the familiar image of a solution of the Mathieu equation for high q values. The example of lithium has been chosen to obtain a steep trap with high oscillation frequencies. The second colum of Table 1 shows, using the cesium atom as an example, that the relatively easily polarisable alkali atoms can also be trapped at lower voltages. In the experiment by Cornell *et al.* on the magnetodynamic trapping of cesium [10] oscillation frequencies of 4-8 Hz and a trap depth of 12 μK have been achieved at $\omega/2\pi = 60$ Hz with 100 G field amplitude. These values can easily be surpassed with the proposed electrodynamic trap. The axial oscillation frequencies ω_z given in Table 1 have been determined from the numerical simulation. They agree well with the prediction of the adiabatic approximation, except in the case of lithium, where q_z is too high for this approximation to give precise results. The radial frequency is always lower than the axial frequency by a factor of two. As a measure of the trap depth the value of the pseudopotential $\Psi(0,0,z_0/2)$ is included in the table. The last row of the table contains the numerically calculated maximal velocity an atom may have at the center of the trap in order to remain on a stable trajectory and not hit the electrodes. This quantity is important to estimate the possibility of loading the trap with cold atoms. For lithium the value corresponds to five times and for cesium to twenty times the respective recoil velocity $\hbar k/m$ for laser cooling on the D_2 line. This shows that the hexapole trap can readily be loaded with cold alkali atoms from a magnetooptical trap or optical molasses. The trap parameters for hydrogen (a light atom of low polarisability) and silver (average mass

Table 1. Possible experimental parameters for hexapole traps for lithium, cesium, hydrogen and silver. Atomic polarisabilities α from ref. [16]. A distance $2z_0 = 2$ mm between the axial electrodes has been assumed in all examples. $v_{\max}$: maximal velocity of a stably trapped atom at the center of the trap.

	^{7}Li	^{133}Cs	^{1}H	^{107}Ag
α [10^{-39} Jm^2/V^2]	2.70	6.63	0.0742	0.87
U_1 [kV]	8	4	8	8
U_2 [V]	10.5	163	55.0	50
U_{30} [kV]	4	2	4	4
$\omega/2\pi$ [Hz]	800	200	450	150
q_z	0.884	0.458	0.535	0.53
$\omega_z/2\pi$ [Hz]	308	32	88	28
$\Psi(0,0,z_0/2)/k_B$ [μK]	260	87	4	50
$v_{\max}$ [m/s]	0.40	0.076	0.21	0.061

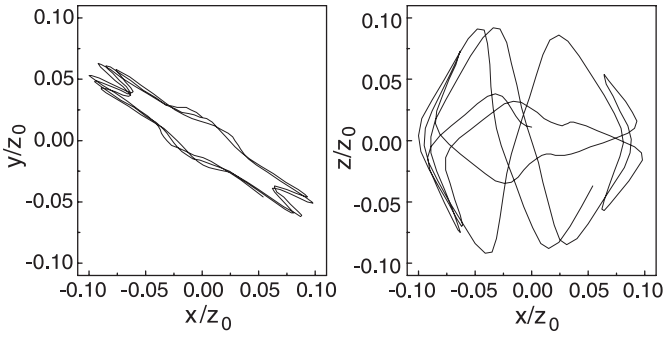

Fig. 2. Trajectory of an atom in the hexapole trap, projected on the xy- and the xz-planes, respectively (parameters for ^{7}Li from Table 1).

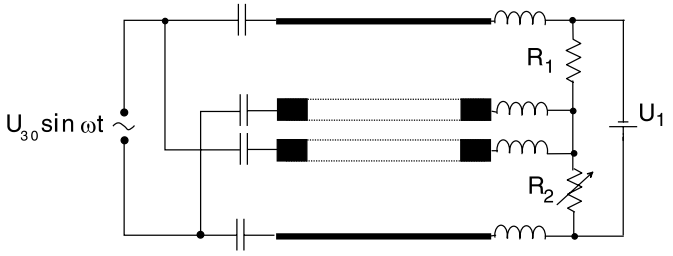

Fig. 3. Practical electrode configuration for the hexapole trap, consisting of two disks and two rings (rotationally symmetric around the vertical axis). The oscillating hexapole potential is coupled capacitively to keep the two high voltage supplies for U_1 and U_3 separated. Radii and distances have been chosen such that the alternating potential has no linear contribution (distance between the disks = disk diameter = $2z_0$, thickness of the rings = distance between the rings = $0.2z_0$, inner ring diameter = $1.45z_0$). The voltage divider R_1/R_2 serves to generate the quadrupole potential U_2 by means of a small shift of the DC potential of the rings with respect to the dipole potential.

and average polarisability), that are included in the table show that these less easily polarisable atoms can be trapped too. However, the very cold atoms required to load the trap are not so easily available for these species. The choice of trapping parameters is simply given by the requirement for q_z to stay within the range 0.4-0.9. The parameters can easily be rescaled: for example, z_0, U_1 and U_3 can be reduced by a common factor while ω is increased by the same factor.

For a practical realisation of the hexapole trap one can not use the electrode setup shown in Figure 1, since the static homogeneous field would be strongly shielded by the endcaps of the hexapole. Figure 3 shows a simple arrangement consisting of only four rotationally symmetric electrodes (two disks and two rings) that generate all three field components: dipole, quadrupole and hexapole. Radii and distances (cf. caption of Fig. 3) have been chosen such that the oscillating part of the potential does not possess a linear term, and consequently does not contribute to the homogeneous field. The lowest order and dominating contribution in a multipole expansion of this potential will then be the hexapole. Since the electrodes do not exactly match the equipotential surface of a hexapol (cf. Fig. 1) the potential also contains small contributions from odd higher order multipoles. For storage close to the center of the trap these contributions are not critical and the results derived above for the pure hexapole potential remain applicable. The static dipole field, applied between the two disks, is slightly shielded by the two rings in the center of the trap. This has to be compensated for by a 20% increase in U_1. To generate the quadrupole component, the two rings are connected via inductances to put them on a common static potential. Using an adjustable voltage divider R_1/R_2, the DC potential of the rings can be shifted somewhat out of the center of the dipole potential. This creates the small quadratic contribution of the quadrupole potential. The whole configuration has been designed to keep the peak field strength between the electrodes below 100 kV/cm for the parameters of Table 1, in order to avoid electrical breakdown.

Note that the stability of the trap is independent of the sign of the polarisability α. This means that low field seekers can be trapped as well as high field seekers. One can even change the modulus or the sign of α while the atoms are trapped. This opens the possibility to induce transitions between energy levels that have different polarisabilities. As long as the lifetime of the excited state is much smaller than the period of the trapping field and as long as the population of this state remains small compared to that of the ground state, the stability of the trap remains unaffected, even if the q_z parameter of the excited state lies outside the stability limit. If these conditions are not fulfilled, the particles are transferred nonadiabatically between two pseudopotential wells of different depth and can gain or lose energy during these transitions. Especially if the sign of α changes, the phase relation between the movement of the atom and the alternating field is disturbed and energy is transferred from the time-dependent trap potential to the kinetic energy of the atoms. In the absence of cooling, this process will finally heat the atoms out of the trap. However, since optical transitions can be induced, it should be possible to apply laser cooling methods [1] like polarisation gradient cooling or Raman cooling and damp the motion of the trapped atoms. As the Zeeman sublevels keep their degeneracy in the purely electrical hexapole trap, these methods are well applicable.

Some cooling of the trapped atoms will also be required to trap a larger number of atoms at high density. Like electrodynamic ion traps, the hexapole trap is a conservative trap only for noninteracting particles or for a single particle. In the presence of collisions between the trapped atoms, these will gain kinetic energy from the oscillating trap potential [9], a phenomenon well known as radiofrequency heating in ion traps [15]. Fortunately, this heating mechanism is much weaker for neutral atoms with their small collision cross-sections than for ions that interact via the long range Coulomb force. At the typical density in a magnetooptical trap (some $10^{10}/\mathrm{cm}^3$) the collision rate is only a few Hz, so that already a weak cooling power of a few μK/s will be sufficient to stabilize the atoms.

I thank J. Reichel, C. Salomon and H. Walther for helpful discussions.

References

1. For a recent review see: C.S. Adams, E. Riis, Prog. Quant. Electr. **21**, 1 (1997).
2. S. Chu, in: *Proceedings of the International School of Physics "Enrico Fermi", Course CXVIII*, edited by E. Arimondo, W. Phillips (North Holland, Amsterdam, 1991) p. 239.
3. W. Phillips, *ibid.*, p. 289.
4. W. Wing, Progr. Quant. Electr. **8**, 181 (1984).
5. W. Ketterle, D.E. Pritchard, Appl. Phys. B **54**, 403 (1992).
6. W. Wing, Phys. Rev. Lett. **45**, 631 (1980).

7. A.L. Migdall, J.V. Prodan, W.D. Phillips, T.H. Bergeman, H.J. Metcalf, Phys. Rev. Lett. **54**, 2596 (1985).
8. J. Söding, D. Guéry-Odelin, P. Desbiolles, G. Ferrari, J. Dalibard, Phys. Rev. Lett. **80**, 1869 (1998).
9. R. Lovelace, C. Mehanian, T. Tommila, D. Lee, Nature **30**, 318 (1985).
10. E. Cornell, C. Monroe, C. Wieman, Phys. Rev. Lett. **67**, 2439 (1991).
11. F. Shimizu, M. Morinaga, Jpn J. Appl. Phys. **31** , L1721 (1992).
12. M. Morinaga, F. Shimizu, Laser Physics **4**, 412 (1994).
13. F. Shimizu, in *Atomic Physics 13*, edited by H. Walther, T. Hänsch, B. Neizert (AIP, New York, 1993).
14. L.D. Landau, E.M. Lifshitz, *Mechanics*, 3rd ed. (Pergamon, Oxford, 1976) § 30.
15. R. Blümel, C. Kappler, W. Quint, H. Walther, Phys. Rev. A **40**, 808 (1989).
16. T.M. Miller, B. Bederson, *Advances in Atomic and Molecular Physics 13*, edited by D.R. Bates, B. Bederson (Academic, New York, 1977).